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| EXAMEN PARCIAL DE RESISTENCIA DE MATERIALES, ELASTICIDAD Y PLASTICIDAD | APELLIDOS.................................................. |
| 2 de febrero de 2012 | NOMBRE...................................................... |
| | CURSO 3º |

Ejercicio 4

En el pórtico ABCD de la figura existe un tirante entre A y C. La rigidez de una sección del pórtico es, $EI = 8043 \text{ kNm}^2$ y del tirante, $E_T A = 84000$ kN.

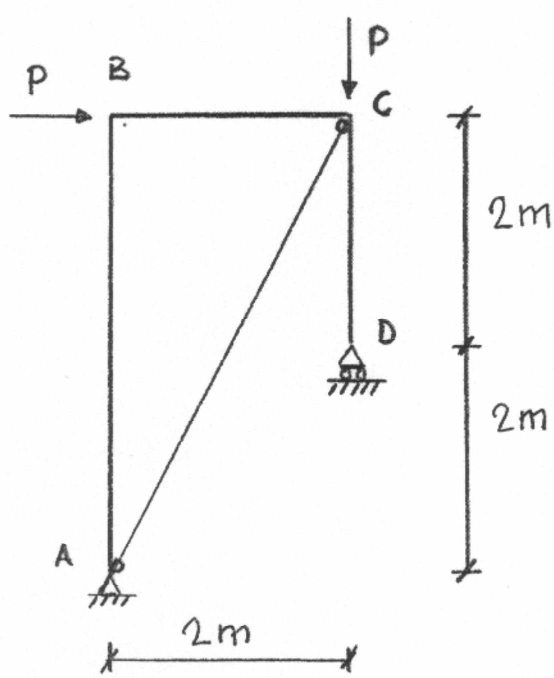
Cuando en el pórtico actúan las cargas indicadas, se pide:

1) Esfuerzo en el tirante AC. (1.4 puntos)

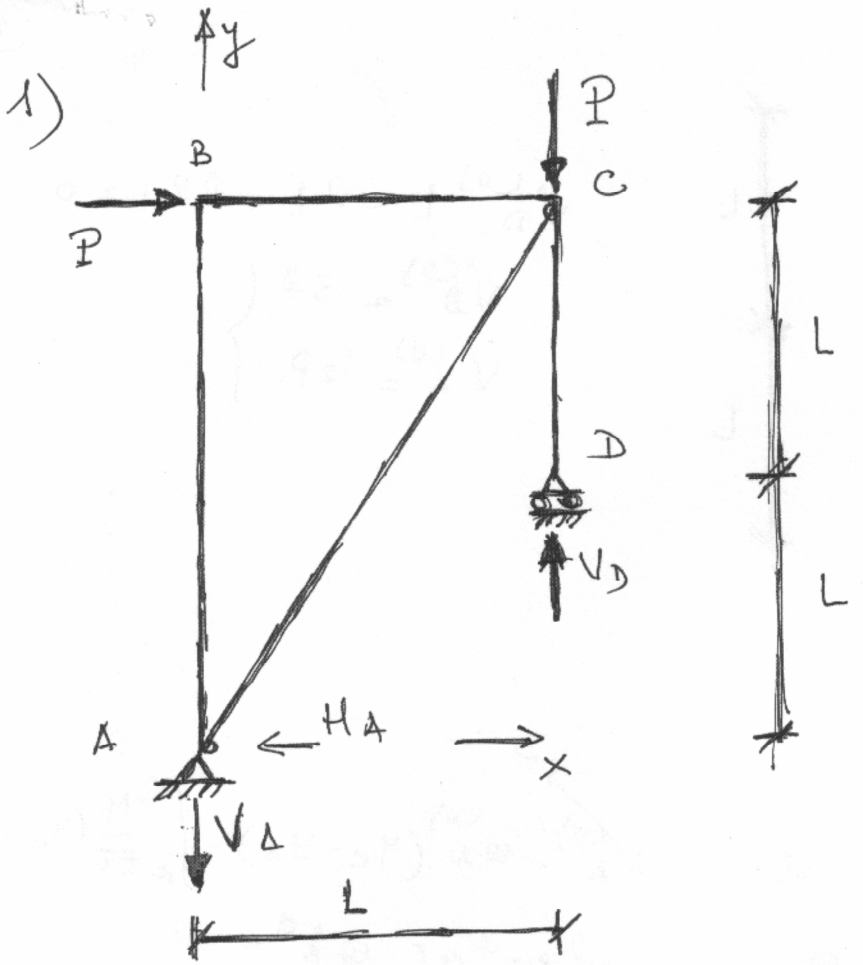
2) Dibujar la ley de momentos flectores en el pórtico. (0.6 puntos)

3) Dibujar a estima la deformada del pórtico. (0.5) puntos)

P = 15 kN



1)



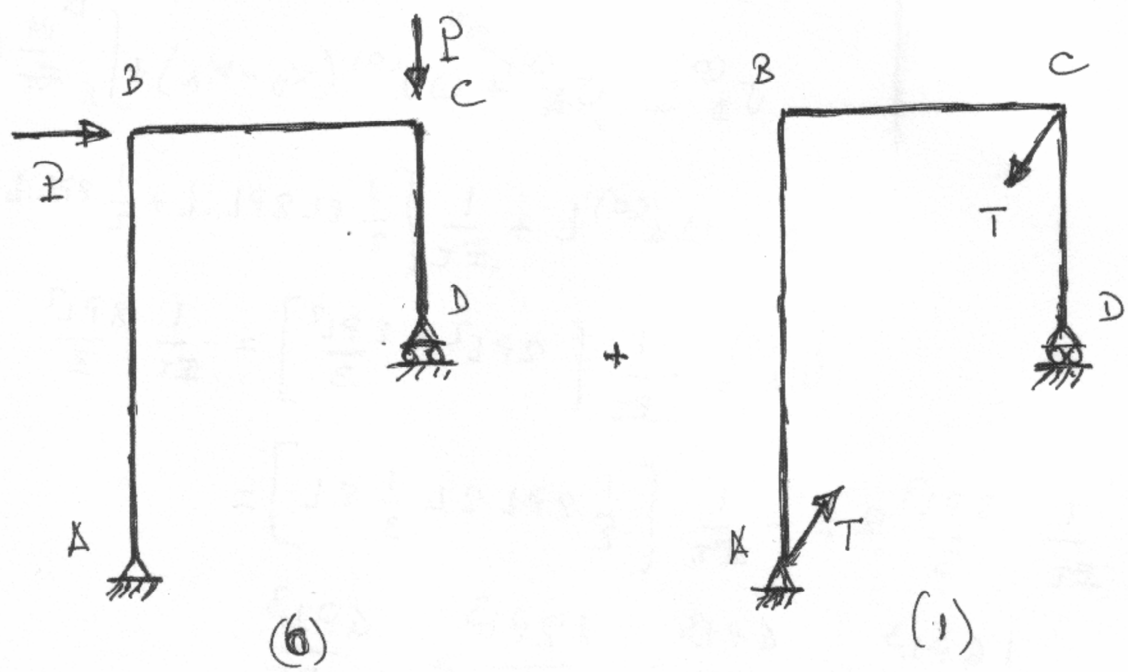

EI (pórtico)
$E_T A$ (tirante)

$H_A = P$

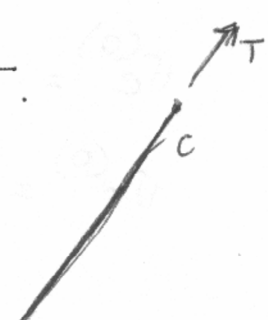
$V_D L - PL - P2L = 0$

$\left.\begin{aligned} V_D &= 3P \\ V_A &= 2P \end{aligned}\right\}.$

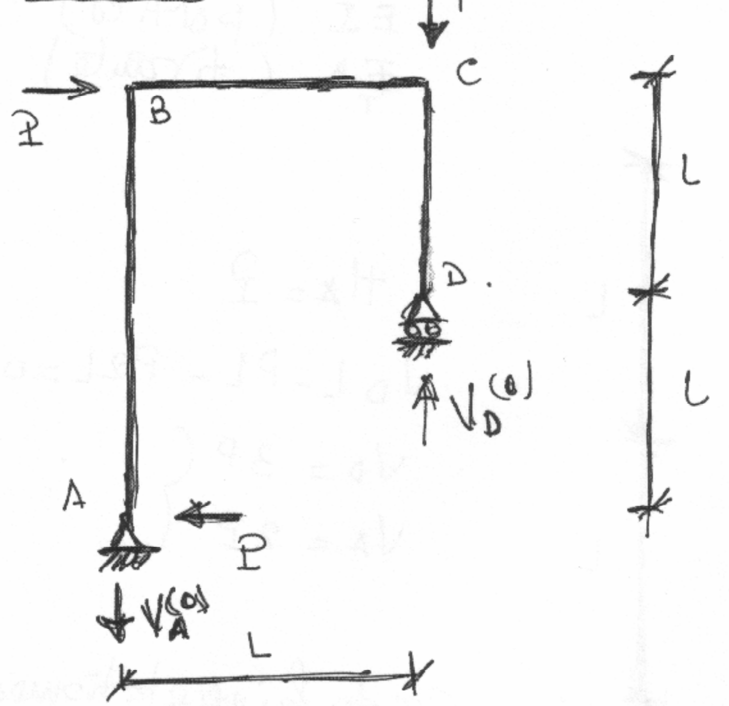
Grado hiperestatismo = 1.




$\Delta AC$ en la estructura ((0) + (1)) =
$\Delta$ longitud del tirante sometido a T.



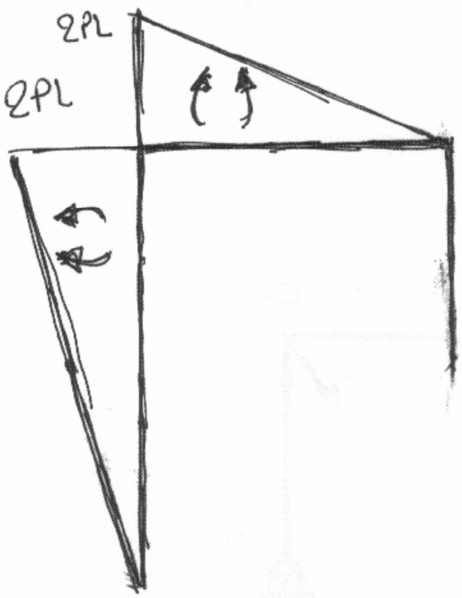
ESTADO (0)




$$V_D^{(0)} L - P L - P\,2L = 0$$

$$V_B^{(0)} = 3P$$

$$V_A^{(0)} = 2P.$$




$$u_C^{(0)} = \cancel{u_A^{(0)}}^{\,0} - \omega_A^{(0)}(y_C - y_A) - \int_A^C \frac{M}{EI}(y_C - y)\,ds$$

Para calcular $\omega_A^{(0)}$:

$$v_D^{(0)} = \cancel{v_A^{(0)}}^{\,0} + \omega_A^{(0)}(x_D - x_A) + \int_A^D \frac{M}{EI}(x_D - x)\,ds$$

$$0 = \omega_A^{(0)} L + \frac{1}{EI}\left[\frac{1}{2}\,2L\,2PL\cdot L + \frac{1}{2}\,2PL\cdot L\,\frac{2}{3}L\right]$$

$$\omega_A^{(0)} = -\frac{1}{EI}\left[2PL^2 + \frac{2PL^2}{3}\right] = -\frac{1}{EI}\,\frac{8PL^2}{3}$$

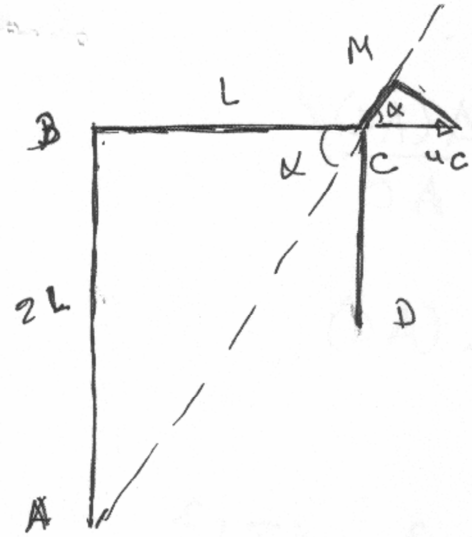
$$u_C^{(0)} = \frac{1}{EI}\,\frac{8PL^2}{3}\,2L - \frac{1}{EI}\left[\frac{1}{2}\,2PL\,2L\,\frac{1}{3}\,2L\right] =$$

$$= \frac{16PL^3}{3EI} - \frac{4PL^3}{3EI} = \frac{12PL^3}{3EI} = \frac{4PL^3}{EI}.$$

$$v_C^{(0)} = \cancel{v_A^{(0)}}^{\,0} + \omega_A^{(0)}(x_C - x_A) + \int_A^C \frac{M}{EI}(x_C - x)\,ds$$

$$v_C^{(0)} = -\frac{8PL^2}{3EI}L + \frac{1}{EI}\left[\frac{1}{2}\,2PL\,2L\cdot L + \frac{1}{2}\,2PL\,L\,\frac{2}{3}L\right] =$$

$$-\frac{8PL^3}{\ } + \frac{1}{\ }\left[2PL^3 + \frac{2PL^3}{\ }\right] = 0$$



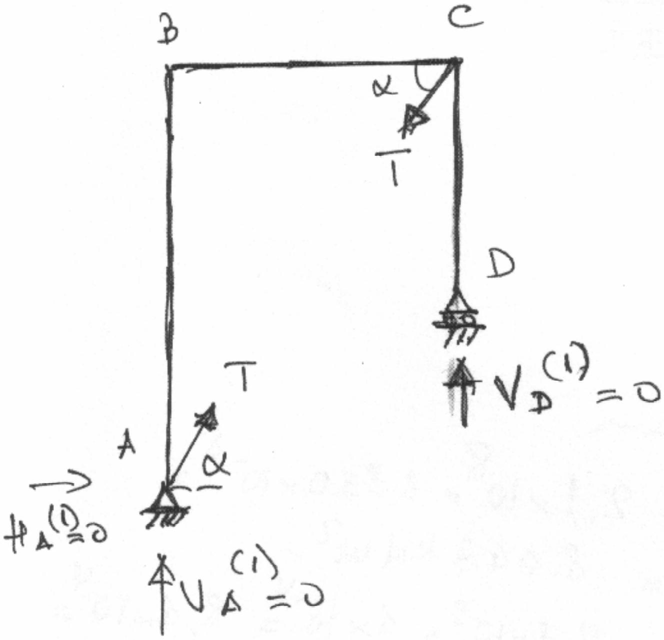

$$CM = u_c \cos\alpha$$

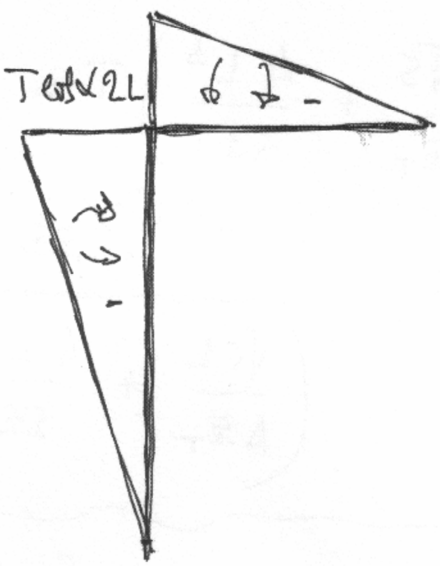
$$AC = \sqrt{4L^2+L^2} = L\sqrt{5}$$

$$\cos\alpha = \frac{L}{L\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$CM = \frac{4PL^3}{EI}\frac{1}{\sqrt{5}}$$

ESTADO (1)



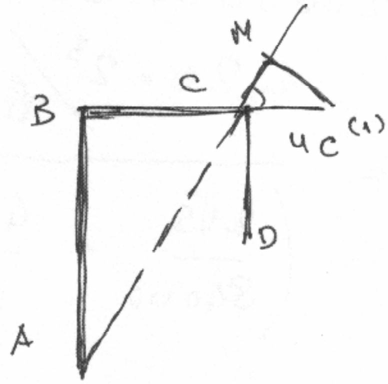




Para calcular $u_c^{(1)}$ y $v_c^{(1)}$ sustituimos P por $T\cos\alpha$ en las expresiones $u_c^{(0)}$ y $v_c^{(0)}$:
(cambiando el signo)

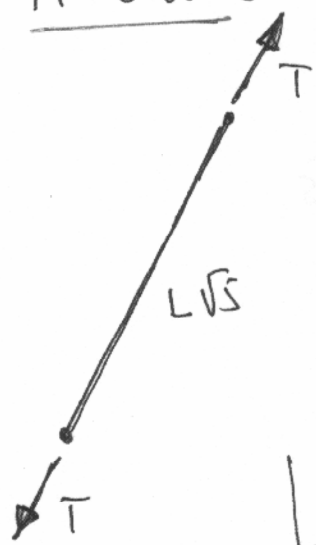
$$u_c^{(1)} = -\frac{4(T\cos\alpha)L^3}{EI} \quad ; \quad v_c^{(1)} = 0.$$

$$u_c^{(1)} = -\frac{4T\frac{1}{\sqrt{5}}L^3}{EI} \quad ;$$

$$CM = u_c^{(1)}\cos\alpha = -\frac{4TL^3}{5EI} = -\frac{4TL^3}{5EI}$$



# Tirante




$$\frac{T}{A} = E_T \frac{\Delta(AC)}{AC}$$

$$\frac{T \cdot L\sqrt{5}}{A E_T} = \Delta(AC)$$

Luego
$$\frac{T L\sqrt{5}}{A E_T} = \frac{4PL^3}{\sqrt{5}EI} - \frac{4TL^3}{5EI}$$

$$\left(\frac{L\sqrt{5}}{A E_T} + \frac{4L^3}{5EI}\right)T = \frac{4PL^3}{\sqrt{5}\cdot EI}$$

$$T = \frac{4PL^3/\sqrt{5}\cdot EI}{\left(\frac{\sqrt{5}L}{A E_T} + \frac{4L^3}{5EI}\right)}$$

Si
$L = 2\,m$
$E = 2{,}1\times10^8\,kN/m^2$
$E_T = 2{,}1\times10^8\,kN/m^2$
$I = 3830\,cm^4$
$A = 4\,cm^2$

$EI = 2{,}1\times10^8 \times 3830\times10^{-8} =$
$= 8.043\,kN\,m^2.$
$E_T A = 2{,}1\times10^8 \times 4\times10^{-4} = 8{,}4\times10^4 =$
$= 84000\,kN$

$P = 20000\,N = 20\,kN$

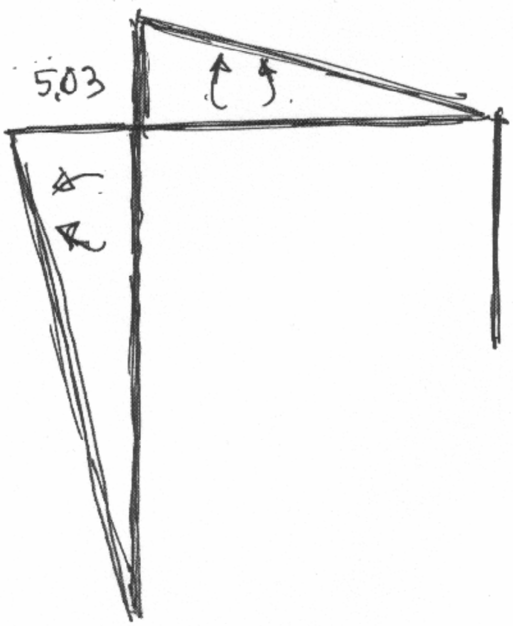
$$T = \frac{4\cdot 20\cdot 2^3/\sqrt{5}\cdot 8043}{\left(\frac{2\sqrt{5}}{84000} + \frac{4\cdot 2^3}{5\cdot 8043}\right)} = \frac{0.035586}{0{,}000053 + 0{,}000796} =$$
$$= \underline{41{,}915194\,KN}$$

## Ley de momentos

2)

$$2PL - T\,\mathrm{en}\,\alpha\, 2L = 2 \times 20 \times 2 - 41{,}91 \times \frac{1}{\sqrt{5}}\, 2 \times 2 =$$

$$= 80 - 74{,}97 = 5{,}03 \text{ mkN}.$$




3) Deformada a estima: